FRACTURE FREQUENCY, POROSITY AND PERMEABILITY.

METHODOLOGY FOR COMPUTING

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1. FREQUENCY.

The fracture frequency we considered hereafter, refer to the number of fracture counted along a one meter line perpendicularly to a considered set of "parallel" fractures. Such a set will also be called a fracture family. This notion is also known as spacing, the distance (perpendicularly) between two successive fractures (of a same set).

In complement we call density a counting along a line of all fractures regardless of their orientation.

The Figure 1 illustrate the calculation, in the case of a core with a specific diameter with a measured fracture density for each fracture family.

The mathematic formulation is:

In the case of a borehole of \(N\) meter long, affected by a number of \(I\) different fracture families,

\[
\begin{align*}
Z_n &= \text{Depth [m] of the nth meter of the borehole for which the frequency is calculated} \\
1 &= n < N \\
F_{in} &= \text{Frequency of the ith fracture family at the depth } Z_n, \quad 1 < i < I \\
d_{in} &= \text{density [fract/m] of the ith fracture family (number of fractures counted each meter)} \\
L &= \text{discretization length [m]} \\
\alpha_i &= \text{angle between the fractures of the ith fracture family and the bore hole} \\
D &= \text{Core section (diameter) [m]}
\end{align*}
\]

The investigation length is:

\[
l_{in} = L \sin \alpha_i + D \cos \alpha_i
\]

and the frequency will be given by:

\[
F_{in} = \frac{d_{in} L}{l_{in}}
\]

The angle \(\alpha\) is given by:

\[
\cos(\pi/2 - \alpha) = \sin \alpha = \frac{\langle V_1, V_2 \rangle}{N(V_1) N(V_2)} \quad \text{with } V_1 \text{ vector parallel to the bore hole and } V_2 \text{ normal vector to the fracture plane; } N(V_1) = \text{Normal of vector } V_1; \quad \langle \rangle \text{ vectorial product.}
\]

if the borehole is vertical, \(\alpha = \pi/2 - \text{Dip}\)
2. POROSITY

2.1. Average opening of a fracture family.

In the case of the open fractures, the decompression from depth stress conditions is taken in account. When the core is lift up to the surface, the decompression has tendency to exaggerate the fractures openings, while they are in reality more close in reservoir conditions.\(^1\) To correct this opening, an empirical factor is applied to the values:

- 0.5 for an opening over 0.5 mm.
- 0.15 for an opening over 1 mm.

This empirical value is based on some informal discussions with reservoir engineers who try to correlate the opening of fractures with the measured permeability.

The distribution of the voids in the fracture planes are not homogenous. Considering the relative small core section in regard with the fracture plane surface, when a fracture is recorded as non open on the core it **doesn't mean that all it's surface is closed.** May be a few centimeter around the core a void could exist.

Several research are actually concerned with this problem, but unfortunately, the distribution of the voids along the fracture surface stay largely unknown.

In consequence for the calculation of the "average opening" of a family, **all the fractures of a family** are used, regardless if they are recorded as open or not on the core. By this way, the voids located near the drill hole are indirectly taken in account.

This is justified because it is an admitted fact that in a fracture family, all the planes have similar characteristics.

The Figure 2 is an example where a bore hole intersects 9 fractures belonging to one family. Only 4 of them present a measurable opening. The final calculation of the average opening is done by cumulating all the measured opening (4 ) and divided them by the total number of fractures cut by the bore hole (9).

2.2. Computing the porosity.

The Figure 3 illustrate the calculation. For simplification, only three fracture families with a similar frequency of 2 fract/m are considered. Each of the three families has a specific average opening from which for a surface of one square meter a **void volume** can be calculated as shown on the upper part of the Figure 3.

These **void volumes** refer to the unit **cubic volumes** of rock cut by the bore hole at a **depth** where the different **fracture frequencies** have been computed. Their spatial orientation is function of that of the considered fracture families, as shaped on the upper part of the Figure 3.

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\(^1\) The correction is not applied to the "bridge" filled fractures, because the filling prevent almost any changes of the opening.
The **global porosity** is calculated by the cumuli of the voids computed above and refer to "**pseudo cubic meter of rock**" formed by the combination of the three upper blocks as shaped on the bottom part of the figure.

The mathematic formulation will be as following:

In the case of a bore hole of \( N \) meter long, affected by a number of \( I \) different fracture families and the following features:

\[
Z_n = \text{Depth [m] of the center of the } n^\text{th} \text{ cubic meter cut by the bore hole for which the porosity is computed } 1=<n=<N.
\]

\( F_{in} = \text{Frequency of the } i^\text{-th fracture family, } 1=<i=<I, \text{ at the depth } Z_n. \)

\( V_{in} = \text{Void volume [m}^3\text{] related to the } i^\text{-th fracture family, at the depth } Z_n. \)

\( e_{in} = \text{Effective average opening [m] of the } i^\text{-th fracture family, at the depth } Z_n. \)

For simplification, the **effective opening of each family is taken as constant along the bore hole length** but of course for more accurate results this average opening should be computed distinctly along the borehole. If not, what is the case generally, the second subscript \( n \) becomes meaningless and only \( e_i \) is used.

\( S = \text{Unit surface of the fracture, } 1 \text{ m}^2 \)

\( P_n = \text{Porosity [%] of the } n^\text{-th cubic meter cut by the bore hole at the depth } Z_n. \)

Then the void volume related to one family will be given by:

\[
V_{in}=F_{in} X e_i X S
\]

and the total porosity is given by:

\[
P_n = \Sigma V_{in} \text{ [% of } 1 \text{m}^3]\]

### 3. PERMEABILITY COMPUTING.

The computing which follow here must be taken only as an indication of the reality. This fact is mostly due to the cubic relation between the permeability and the fracture opening which can only be roughly measured on cores. But the reader must keep in mind that this computation is done as a quick outlook of the combined influence of both fracture frequencies and opening on the flowing.

He should not compare it with a heavy and sophisticated method of fracture modeling which take in account the intersections and "the probable" fracture length and aperture as it can be approach by "FRACMAN" software for instance. It constitute a "guide" for a decision of a drilling orientation which must be taken unfortunately in a hurry.
All the explanation given for the porosity computing concerning the fracture frequency and the average fracture family opening stay used here.

3 papers written by KIRALY 1969, 1971, 1978\(^2\) are used. The equations used to calculate the fracture permeability tensor are partially detailed in the paper of 1971, but many more details and example are given in the last paper of 1978, so please refer to them for specialized information.

**For oil:**

We considered as explained by Kiraly that the flow obeys to the Poiseuille law. The codes used to express the flow in a single fracture are given by WITTKE 1968\(^3\), the generalization to several fracture families is given by KIRALY.

Generally we give the geometric permeability tensor which is only dependant from the geometrical aspect of the reservoir, but if asked we can introduce the fluid viscosity in order to give the hydraulic conductivity.

**For gas:**

Here the Poiseuille law should not be used (high velocity) and the compression of gas should be introduced. But to avoid the use of the compressibility we approximate the gas in down condition as a liquid where the flow doesn’t obey to the Poiseuille law.

We assume that the pressure do not vary so much in a rock volume of one cubic meter and so we neglect the compressibility of the gas (it doesn’t change so much the results in regard with the influence of the uncertainty on the opening).

(WITTKE 1968) give some formula for non laminar or non parallel flowing for water in fracture plane. We choice a transition zone formula for a non parallel flowing but which stay laminar (the velocity remain not so high in small fractures): the LOMIZE. We choice a high rugosity coefficient in order to minimize the velocities.

**REMARK**

Of course again we say that the given values must be considered just as an idea of the real permeability values, but the relative distribution of the value along the bore hole is meaningful.

Same the angle values of the distribution of the permeability vector K1, K2, K3 well represent the anisotropy of the fractured media. So the "best well" orientation which is given, is reliable for an efficient "reservoir" draining as long we keep in mind that these calculations have been done with the respect of their limitations hypothesis announced above.

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Fracture repartition for one family

For the calculation, the fractures are supposed to have a length of 1 meter

\[ F = \frac{d \cdot L}{l} \]

\( d \) is the density of fractures and:
\[ l = L \sin \alpha + D \cos \alpha \]

\( \alpha \) is the angle between the fractures and the borehole, if the borehole is vertical \( \alpha = \frac{\pi}{2} - \text{Dip} \)

But in general case the formula to calculate the angle \( \alpha \) is:

\[ \cos(p/2-\alpha) = \sin \alpha = \frac{V_1 \cdot V_2}{|V_1| \cdot |V_2|} \]

with \( V_1 \) vector parallel to the borehole and \( V_2 \) normal vector to the fracture plan. \( |V_1| = \text{Norm of vector } V_1 \), \( \cdot \) vect. product.

In the picture example: The borehole is vertical, the fracture family dipping is 80°, \( \alpha = 10° \), \( L = 1 \text{ m} \), \( D = 0.1 \text{ m} \), \( d = 2 \text{ fr/m} \) then
\[ l = 1 \cdot \sin 10° + (0.1) \cdot \cos 10° = 0.27 \]
and
\[ F = 2/0.27 \approx 7.3 \text{ fr/m} \]
OPENING HYPOTHESIS

The opening of a fracture is not constant along the whole fracture's length. The average opening of a fracture family is calculated from a drill core by cumulating the opening of all fractures belonging to one family and dividing this result by the number of measured fractures.

Even the fractures with an unmeasurable opening on the core are used in order to take into account the possible openings located in 0.5 meter around the well.

The drill hole illustrated here intersects 9 fractures belonging to one family. Only four of them have a measurable opening. The average opening is:

\[
\frac{(0.4+0.1+0.6+0.2)}{9}
\]
Fam.1 effective open = 0.2mm = 2.10^{-4} m  
Fam.2 effective open = 0.3mm = 3.10^{-4} m  
Fam.3 effective open = 0.4mm = 4.10^{-4} m

For each family: void volume for 1 m³ = Frequency × effective opening × 1 m³  
All the frequencies are taken for simplification equal to 2fr/m.

\[
\text{POROSITY} = \frac{\text{Fam.1 void volume} + \text{Fam.2 void volume} + \text{Fam.3 void volume}}{1m^3}
\]

\[
\text{porosity} = \frac{(2.10^{-4}) \times 2 \times 1 + (3.10^{-4}) \times 2 \times 1 + (4.10^{-4}) \times 2 \times 1}{1} = 0.18\%
\]

The porosity results worth for the specific volume delimited by the 3 unit blocs (1m section) centered at the well depth where the fractures frequencies are computed.
Two fracture families are considered with the same opening and frequencies, the ellipsoid of permeability has a big axis almost parallel to the intersection of the two fracture families, $K_1$ and $K_2$ are different.

Case with one family of fracture, the ellipsoid becomes a disk parallel to the fracture. The position of $K_1$ and $K_2$ is not defined on this disk, the flowing is uniform in this fracture plane.
EXAMPLE OF RESULTS FROM CASE HISTORY

Why calculate the true frequency

The rose diagram give a wrong image of the reality. The family F1 seems to be dominant, while it's a secondary direction in comparison with F2.

Thus it's important to compute frequency before stating a geological conclusion.
Fractures Frequency

Fractures measured: 57
Fractures calculated: 131.8

Fractures measured: 18
Fractures calculated: 70.4

Fractures measured: 17
Fractures calculated: 77.0

Fracture frequency (calculated)
Fracture density (measured)
Density stereonet
Azimuth distribution of K1, K2 and K3 vectors

Mean lineation vector
N278°, 85°W

Mean lineation vector
N78°, 2°E

Mean lineation vector
N162°, 1°S

Schmidt net-lower hemisphere